Vector mesons in nuclear medium with small three momentum, a QCD sum rule approach

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Using the QCD Operator Product Expansion, we derive the real part of the transverse and longitudinal vector vector correlation function with the ρ, ω quantum numbers to leading order in density and in $\mathbf{q}^2$ at $-\omega^2 \to \infty$. To dimension 6, only twist-2 and 4 operators contribute. These OPE, through the energy dispersion relation, provide model independent constraints for the $\mathbf{q}$ dependence of the vector meson spectral density in nuclear medium. We further make a QCD sum rule type of analysis to extract the momentum dependence of the vector meson dispersion relation in medium. The contributions from twist-2 operators are added up to infinite order to check the validity of the OPE at the relevant Borel window.

1. Introduction

The properties of vector meson in nuclear medium have been the focus of current interest due to its potential role to provide one with a direct observable of the nuclear medium effects, associated with chiral symmetry restoration, via dileptons in H-A or A-A reactions[1]. Indeed dileptons from Relativistic Heavy Ion Collisions (RHIC)[2] seemed to suggest a non-trivial change of the vector meson spectral density in a hot/dense environment, which can be understood in terms of model calculations[3] based on decreasing vector meson masses in hot/dense medium[4,5]. However, model calculations[6,7] based on changes of the vector meson spectral densities using hadronic variables also seem to explain the main features of the CERES data. In all of the approaches, the central question is, how the spectral density changes in hot/dense matter[8]. In this talk, I will provide constraints on the three momentum ($\mathbf{q}$) dependence of the vector meson spectral density in nuclear medium[9,10]. The approach is based on the Operator Product Expansion (OPE) in QCD and provides model independent constraints that can be used to check the validity of any model calculation. In the second stage, I will make a simple ansatz for the spectral density near the vector meson mass and look at what the parameters of the ansatz should be to satisfy the constraints[9].

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2. OPE

Consider the correlation function of the vector current $J_\mu = \bar{q}\gamma_\mu q$ in nuclear matter;

$$\Pi_{\mu\nu}(\omega, \mathbf{q}) \;=\; i\int d^4x e^{iqx}\langle G|T[J_\mu(x)J_\nu(0)]|G\rangle. \tag{1}$$

Here $|G\rangle$ is the nuclear ground state at rest and $q = (\omega, \mathbf{q})$. In what follows, when we give result for explicit vector meson, we will use the currents $J_\mu^{\rho,\omega} = \frac{1}{2}(\bar{u}\gamma_\mu u \mp \bar{d}\gamma_\mu d)$ for the ρ, ω mesons.

In general, because the vector current is conserved, the polarization tenser in eq.(1) will have only two invariant functions[11].

$$\Pi_{\mu\nu}(\omega, \mathbf{q}) = \Pi_T q^2 \mathrm{P}^T_{\mu\nu} + \Pi_L q^2 \mathrm{P}^L_{\mu\nu}, \tag{2}$$

where we assume the ground state to be at rest, such that, $\mathrm{P}^T_{00} = \mathrm{P}^T_{0i} = \mathrm{P}^T_{i0} = 0$, $\mathrm{P}^T_{ij} = \delta_{ij} - \mathbf{q}_i\mathbf{q}_j/\mathbf{q}^2$ and $\mathrm{P}^L_{\mu\nu} = (q_\mu q_\nu/q^2 - g_{\mu\nu} - \mathrm{P}^T_{\mu\nu})$. When $\mathbf{q} \to 0$, $\Pi_L = \Pi_T$, as in the vacuum.

In this work, we are only interested in the small three momentum ($\mathbf{q}$) dependence of the polarization functions. Therefore we will make a small $\mathbf{q}$ expansion of the correlation function and look at its energy dispersion relation at fixed $\mathbf{q}$,

$$\begin{aligned}\mathrm{Re}\Pi_{L,T}(\omega^2, \mathbf{q}^2) &= \mathrm{Re}\left(\Pi^0(\omega^2, 0) + \Pi^1_{L,T}(\omega^2, 0)\ \mathbf{q}^2 + \cdot\cdot\right) \\ &= \int_0^\infty du^2 \left(\frac{\rho(u,0)^0_{L,T}}{(u^2-\omega^2)} + \frac{\rho(u,0)^1_{L,T}}{(u^2-\omega^2)}\ \mathbf{q}^2 + \cdot\cdot\right),\end{aligned} \tag{3}$$

where $\rho(u, \mathbf{q}) = 1/\pi \mathrm{Im}\Pi^R(u^2, \mathbf{q}^2)$, and R denotes the retarded correlation function. The OPE for Π^0 and its change in nuclear medium together with its corresponding energy dispersion relation provide constraints on the mass shift and spectral changes at $\mathbf{q} = 0$[5]. In this talk, we will discuss the three momentum dependence by studying the OPE and its changes in medium for $\Pi^1_{L,T}$.

In general, the OPE [12,13] for the polarization function at $Q^2 = -\omega^2 + \mathbf{q}^2 \to \textit{large}$ will look as follows.

$$\begin{aligned}\Pi_{\mu\nu}(\omega, \mathbf{q}) \;=\; & (q_\mu q_\nu - g_{\mu\nu}q^2)\left[-c_0 \ln|Q^2| + \sum_n \frac{1}{Q^n} A^{n,n}\right] \\ & + \sum_{\tau=2}\sum_{k=1}[-g_{\mu\nu}q_{\mu_1}q_{\mu_2} + g_{\mu\mu_1}q_\nu q_{\mu_2} + q_\mu q_{\mu_1} g_{\nu\mu_2} + g_{\mu\mu_1}g_{\nu\mu_2}Q^2] \\ & \qquad q_{\mu_3}\cdot\cdot q_{\mu_{2k}} \frac{2^{2k}}{Q^{4k+\tau-2}} A^{2k+\tau,\tau}_{\mu_1\cdot\cdot\mu_{2k}} \\ & + \sum_{\tau=2}\sum_{k=1}[g_{\mu\nu} - q_\mu q_\nu/q^2] q_{\mu_3}\cdot\cdot q_{\mu_{2k}} \frac{2^{2k}}{Q^{4k+\tau-2}} C^{2k+\tau,\tau}_{\mu_1\cdot\cdot\mu_{2k}}.\end{aligned} \tag{4}$$

Here we have pulled out the trivial $\frac{q^{\alpha}\cdot\cdot}{Q^n}$ dependence so that $A^{d,\tau}, C^{d,\tau}$ represents the residual Wilson coefficient times matrix element of an operator of dimension d and twist

$\tau = d - s$, where $s = 2k$ is the number of spin index of the operator. The first set of terms comes from the OPE of scalar operators, the second set from operators with spin, which have been written as a double sum of twist and spin. A, C represents the two linearly independent sum of operators, which reflect the two linearly independent polarization directions.

The longitudinal and transverse polarization functions can be obtained from eq.(4). The $\mathbf{q}$ dependence coming from the first line of eq.(4), namely the contribution from the scalar operators, comes from the $\mathbf{q}$ dependence in Q^2. This forms the so called "trivial" $\mathbf{q}$ dependence, and comes from replacing $\omega^2 \to \omega^2 - \mathbf{q}^2$ when going from zero to finite three momentum. Here, we are not interested in these trivial dependence and will only look at the "non-trivial" $\mathbf{q}$ dependence in $\Pi^1_{L,T}$. Consequently the scalar operators do not contribute to $\Pi^1_{L,T}$. Only operators with spins contribute to the non-trivial $\mathbf{q}$ dependence. A prescription to find the nontrivial $\mathbf{q}^2$ dependence in the OPE is to first calculate the total contribution proportional to $\mathbf{q}^2$ and then subtract out the trivial dependence; $\frac{1}{\omega^n}(d)\frac{\mathbf{q}^2}{\omega^2} \to (d - \frac{n}{2})\frac{\mathbf{q}^2}{\omega^{n+2}}$, and its contribution to Π^1 would be $(d - \frac{n}{2})\frac{1}{\omega^{n+2}}$. Another equivalent method is to express the polarization function in terms of $\Pi(Q^2, \mathbf{q}^2)$ and extract the linear $\mathbf{q}^2$ term.

3. Linear density Approximation

In the linear density approximation,

$$\langle G|A|G\rangle = \langle 0|A|0\rangle + \frac{\rho_n}{2m}\langle p|A|p\rangle, \tag{5}$$

where the first term denotes the vacuum expectation value, which vanishes for operators with spin, and the second term the nucleon expectation value with the normalization $\langle p|p\rangle = 2p_0\delta^3(p - p)$. ρ_n, m denotes the nuclear density and the mass of the nucleon respectively.

As in the vacuum, we will truncate our OPE at dimension 6 operators. This implies that in our OPE in eq.(4), we will have contributions from $(\tau, s) = (2,2), (2,4), (4,2)$. The nucleon matrix elements of the $\tau = 2$ operators are very precisely known. The $\tau = 4$ matrix elements appearing in the ρ, ω sum rule are similar to those appearing in electron DIS[14] and have been estimated [15,16]up to about $\pm$ 30% uncertainty from available DIS data from CERN and Slac.

The final form of the OPE for Π^1 looks as follows.

$$\Pi^1_{L,T}(\omega)/\rho_n = \frac{b_2}{\omega^6} + \frac{b_3}{\omega^8}. \tag{6}$$

For ρ, ω, the transverse (T) and longitudinal (L) parts give,

$$\begin{aligned}
b_2^T &= (\frac{1}{2}C^q_{2,2} - \frac{1}{2}C^q_{L,2})mA_2^{u+d} + (C^G_{2,2} - C^G_{L,2})mA_2^G, \\
b_3^T &= (\frac{9}{4}C^q_{2,4} - \frac{5}{2}C^q_{L,4})m^3A_4^{u+d} + (\frac{9}{2}C^G_{2,4} - 5C^G_{L,4})m^3A_4^G \\
&\quad + \frac{1}{2}m\left(-(1+\beta)(K^1 + \frac{3}{8}K^2 + \frac{7}{16}K^g) + K^1_{ud}(1 \pm 1)\right),
\end{aligned}$$

$$b_2^L = -\frac{1}{2}C_{L,2}^q m A_2^{u+d} - C_{L,2}^G m A_2^G, \tag{7}$$

$$b_3^L = (\frac{1}{2}C_{2,4}^q - \frac{5}{2}C_{L,4}^q)m^3 A_4^{u+d} + (C_{2,4}^G - 5C_{L,4}^G)m^3 A_4^G$$
$$+\frac{m}{8}(1+\beta)\left(K^2 - \frac{3}{2}K^g\right), \tag{8}$$

where $\pm$ refers to the ρ and ω case. Here, for even n,

$$A_n^q = 2\int_0^1 dx x^{n-1}[q(x,Q^2) + \bar{q}(x,Q^2)] \ \ , \ \ A_n^G = 2\int_0^1 dx x^{n-1} G(x,Q^2), \tag{9}$$

where $q(x,Q^2)$ and $G(x,Q^2)$ are the quark and gluon distribution functions. We will use the HO parameterization for these obtained in ref[17] which should be used with the Wilson coefficients $C's$ in the $\overline{\rm MS}$ scheme[18] and given below. Terms proportional

Table 1
Wilson coefficients for $\tau = 2$. $T(R) = \frac{f}{2}, C_2(R) = \frac{4}{3}$.

Wilson coefficient	$n=2$	$n=4$
$C_{2,n}^q = 1 + \frac{\alpha_s}{4\pi}B_{2,n}^{NS}$	$1 + \frac{\alpha_s}{4\pi}(0.44)$	$1 + \frac{\alpha_s}{4\pi}(6.07)$
$C_{L,n}^q = \frac{\alpha_s}{4\pi}C_2(R)\frac{4}{n+1}$	$\frac{\alpha_s}{4\pi}\frac{16}{9}$	$\frac{\alpha_s}{4\pi}\frac{16}{15}$
$C_{2,n}^G = \frac{\alpha_s}{4\pi}T(R)\frac{4}{f}[\frac{4}{(n+1)} - \frac{4}{(n+2)} + \frac{1}{n^2} - \frac{n^2+n+2}{n(n+1)(n+2)}(1+\sum_{j=1}^n \frac{1}{j})]$	$\frac{\alpha_s}{4\pi}(-\frac{1}{2})$	$\frac{\alpha_s}{4\pi}(-\frac{133}{180})$
$C_{L,n}^G = \frac{\alpha_s}{4\pi}T(R)\frac{1}{f}\frac{16}{(n+1)(n+2)}$	$\frac{\alpha_s}{4\pi}\frac{2}{3}$	$\frac{\alpha_s}{4\pi}\frac{4}{15}$

to $K's$ come from $\tau = 4, s = 2$. We will use the set of K values obtained in ref.[15]; (K^1, K^2, K_{ud}^1, K^g) = ($-0.173\mathrm{GeV}^2, 0.203\mathrm{GeV}^2, -0.083\mathrm{GeV}^2, -0.238\mathrm{GeV}^2$) and we take $\beta = 0.5$. For the $\tau = 4$ operators, we neglect the Q^2 dependence.

4. Constraints

We will now make a Borel transformation of ω^2 times eq.(6) and look at the dispersion relation.

$$\rho_n\left(\frac{b_2}{M^2} - \frac{b_3}{2M^4}\right) = \int ds \rho^1(s) s e^{-s/M^2} + b_{scatt}. \tag{10}$$

The reason for multiplying by ω^2 is to get rid of any possible subtraction constants proportional to $1/\omega^2$. There could be still further subtraction constants proportional to b_{scatt}/ω^4, which we included. b_{scatt} should also be calculated in any model calculations. The value from particle-hole intermediate state gives $b_{scatt} = -1/(4m)\rho_n$ for the longitudinal ρ, ω meson and zero for the transverse parts.

After the Borel transformation, the contribution from dimension=n operators in the OPE side is further divided by $(n-2)!$. This will make our truncation at dimension 6 operators valid even to smaller Borel mass region. In fig.1, we have plotted OPE as a function of M^2. The solid line shows the left hand side of eq.(10). The dot-dashed line without

the twist-4 contribution and the long dashed line without dimension 6 contribution. At $M^2_{min} \sim .8\mathrm{GeV}^2$ ($M^2_{min} \sim 2.5\mathrm{GeV}^2$), the contribution of dimension 6 operators become about 40% of the contributions from dimension 4 operators for transverse (longitudinal) polarization. Therefore, the OPE should be reliable only above these minimum values of the Borel mass, where the contributions from higher dimensional operators should be similarly suppressed. To support our argument, we have also plotted (short dashed line) the contributions from adding up the infinite sum of the twist-2 operators. This is possible because for the twist-2 operators, all the moments are know. As can be seen from the figure, once we are above the minimum Borel mass, the difference to the solid line is vanishingly small, confirming the validity of our OPE above the Minimum Borel mass. Now the constraints for the spectral density would be eq.(10) , but applied only above

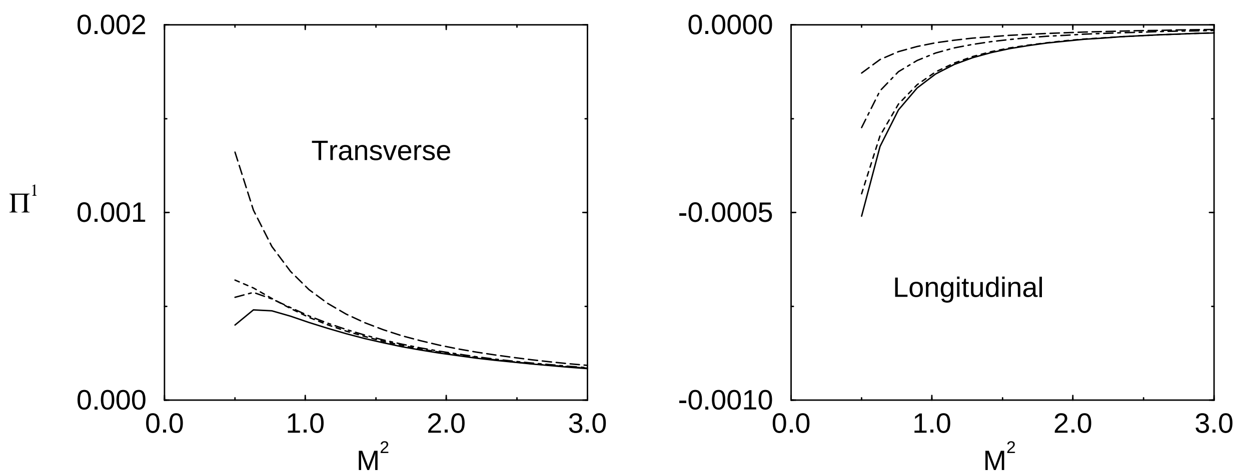

Figure 1. Solid line denotes our OPE, short dashed line with the infinite sum of the twist-2 operators, dashed line OPE without dim 6, dot-dashed OPE without twist-4. All curves are at nuclear matter density and GeV^2 unit for both axis

$M^2 > M^2_{min}$.

As can be seen in eq.(10), the Borel transformation also changes the weighting factor of the spectral density to an $exp(-s/M^2)$. This has the following advantage for practical applications of our constraint. For small values of the Borel mass, the contribution of the spectral density at larger energy is exponentially suppressed. Consequently, in a model calculation, one can concentrate on the change of the spectral density near the vector meson mass region and below and model the higher energy changes with a simple pole like contribution.

4.1. Results with simple ansatz

We now show the result with the following ansatz of the spectral density.

$$\rho(u) = \frac{F\Gamma_\rho(m_\rho + \frac{f}{m_\rho}\mathbf{q}^2)}{(u^2 - m_\rho^2 - a\mathbf{q}^2)^2 + m_\rho^2(\Gamma_\rho^2 + g\mathbf{q}^2)} + c\theta(u^2 - (S_0 + s\mathbf{q}^2)). \tag{11}$$

We expand this to linear order in $\mathbf{q}^2$ to obtain ρ^1 and use the constraints in eq.(10) to obtain the best fit value of the constants f, a, g, s. In the limit $\Gamma \to 0$ and $g = 0$, the results were obtained in ref.[9][2] which translates into the following momentum dependence in the mass. $\frac{m_\rho(\rho_n)}{m_\rho(0)} = 1 - (0.023 \pm 0.007)(\frac{\mathbf{q}}{0.5})^2(\frac{\rho_n}{\rho_0})$, where $\mathbf{q}$ is in GeV unit and ρ_0 is the nuclear matter density. For the ω meson, 0.023 changes to 0.016.

5. Conclusion

We have shown that there are model independent constraints on the three momentum dependence of the vector meson spectral density in nuclear medium. Any model calculations can be checked to see if it satisfies the constraints. Phenomenological models based on p-wave nucleon resonances seems to overestimate the momentum dependence[19].

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